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Notes on Branes in Matrix Theory

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Abstract:

We study the effective actions of various brane configurations in Matrix theory. Starting from the $0+1$ dimensional quantum mechanics, we replace coordinate matrices by covariant derivatives in the large N limit, thereby obtaining effective field theories on the brane world volumes. Even for noncompact branes, these effective theories are of Yang-Mills type, with constant background magnetic fields. In the case of a D2-brane, we show explicitly how the effective action equals the large magnetic field limit of the Born-Infeld action, and thus derive from Matrix theory the action used by Polchinski and Pouliot to compute M-momentum transfer between membranes. We also consider the effect of compactifying transverse directions. Finally, we analyze a scattering process involving a recently proposed background representing a classically stable D6+D0 brane configuration. We compute the potential between this configuration and a D0-brane, and show that the result agrees with supergravity.

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1 Introduction

Matrix theory [1] purports to be a complete, non-perturbative formulation of M theory. Since a great deal is known about the physics of M theory in various corners of moduli space, much effort has recently been directed towards verifying that such physics can be recovered from Matrix theory. In this paper we will be primarily interested in studying the description of D-branes that emerges when Matrix theory is compactified on a circle, yielding the type IIA theory. Some salient features of D-branes in IIA theory which one would like to obtain from Matrix theory include the following: there exist Dp-branes for p = 0,2,4,6,8; the low energy dynamics of k parallel Dp-branes is described by a supersymmetric p+1 dimensional U(k) gauge theory; the (not necessarily low energy) dynamics of a single D-brane is described by a Born-Infeld type action. In addition, one knows the supergravity fields produced by D-branes and so also the corresponding small angle scattering amplitudes of several such objects.

Banks, Seiberg, and Shenker [2], and S.-J. Rey [3], took some first steps towards deriving the effective actions of D-branes from Matrix theory. They identified matrix assignments corresponding to D-branes by identifying the appropriate central charges in the 11 dimensional light cone gauge supersymmetry algebra. Then by analyzing the fluctuations around such backgrounds and replacing large N commutators by Poisson brackets, they were able to recover the general structure of the known effective actions to quadratic order in fields. Here, our goal is similar, but we wish to go beyond quadratic order and to keep careful track of numerical factors. To do so successfully, we found it necessary to replace matrix commutators not by Poisson brackets but rather by the commutators of covariant derivatives, much as one does when considering toroidal compactification. By so doing, we are able to obtain gauge invariant actions which precisely reproduce the fluctuations of the Born-Infeld action in the presence of a large background magnetic field.

There have been a number of studies comparing D-brane scattering amplitudes obtained from Matrix theory and from supergravity. On the Matrix theory side, the proce-

dure involved computing the effective action of a finite N matrix configuration, and then extrapolating to large N. Strictly speaking, this procedure is inconsistent as the backgrounds require that certain finite N matrices have a commutator proportional to the identity, even though such matrices cannot exist. In our approach, we avoid this problem by taking $N \to \infty$ from the outset. As a specific application, we consider the scattering of a D0-brane from a configuration composed purely of D6 and D0 branes [4], which has not been studied before. We find complete agreement at the one loop level between Matrix theory and supergravity.

The remainder of this paper is organized as follows. In section 2 we rewrite the Matrix theory action in a manner suitable for the study of D-branes. Sections 2.1 and 2.2 focus on the effective action for D2-branes, and section 2.3 on the effective action for D4-branes. In section 3 we compute the potential between a D0-brane and a D6+D0 configuration, and in section 4 we discuss some of our results.

2 Fluctuations of branes in Matrix theory

The action governing Matrix theory is gotten by dimensionally reducing the action of 10 dimensional Super Yang-Mills theory to 0+1 dimensions. In string units ($2\pi\alpha' = 1$) the bosonic part of the Lagrangian is[1]

$$\mathcal{L}_B = \mathrm{Tr}\ L_B\ ;\ L_B = \frac{T_0}{2}\{(D_0 X_I)^2 + \frac{1}{2}[X_I, X_J]^2\} \tag{1}$$

where T_0 is the D0-brane "tension"

$$T_0 = \frac{1}{\sqrt{\alpha'} g} = \frac{\sqrt{2\pi}}{g}\ ,$$

the covariant derivative D_0 is

$$D_0 = \partial_t - i[A_0,\ \cdot\ ]$$

[1]following [1], Appendix B

and $I = 1, 2, \ldots, 8, 9$. A p-brane background is described by

$$X_r = U_r \; ; \; r = 1, 2, \ldots, p \tag{2}$$

$$X_i = 0 \; ; \; i = p + 1, \ldots, 9 \tag{3}$$

where $U_r, r = 1, \ldots, p$ are certain matrices to be specified later.

Fluctuations around this background are denoted by A_r, ϕ_i,

$$X_r = U_r + A_r \tag{4}$$

$$X_i = \phi_i \; . \tag{5}$$

Following Banks, Seiberg and Shenker [2], we substitute the above expansions into the Lagrangian L_B, and then regroup and rename the terms so that we arrive at the following form:

$$L_B = \frac{T_0}{2}\{-\frac{1}{2}(F_{\mu\nu})^2 + (D_\mu \phi_i)^2 + \frac{1}{2}[\phi_i, \phi_j]^2 + i[U_r, U_s]F_{rs} + \frac{1}{2}[U_r, U_s]^2\} \; , \tag{6}$$

where $\mu, \nu = 0, 1, \ldots, p; r, s = 1, \ldots, p$ and

$$\begin{aligned} F_{0r} &= -F_{r0} = \partial_0 A_r + i[U_r, A_0] - i[A_0, A_r] \\ F_{rs} &= -i[U_r, A_s] + i[U_s, A_r] - i[A_r, A_s] \\ D_0\phi_i &= \partial_0 \phi_i - i[A_0, \phi_i] \\ D_r\phi_i &= -i[U_r, \phi_i] - i[A_r, \phi_i] \; . \end{aligned}$$

(In the above, $(D_\mu\phi_i)^2 = (D_0\phi_i)^2 - (D_r\phi_i)^2$ etc.)

So far, we have not done anything other than reorder the terms and give them new names. Now we try to interpret the result in the context of some particular p-brane backgrounds.

2.1 Infinite membrane backgrounds (p=2)

The 11 dimensional SUSY algebra admits central charges corresponding to the presence of membranes. Banks, Seiberg and Shenker [2] computed the central charges starting from the Matrix theory action and found[2]

$$Z_{rs} = -\frac{i}{2\pi}\mathrm{Tr}[X_r, X_s] \ . \tag{7}$$

It follows immediately that Z_{rs} vanishes for finite N. On the other hand, it has recently been proposed by Susskind [5] that the finite N version of Matrix theory is to be interpreted as describing a finite longitudinal momentum sector of M theory quantized in light cone gauge. The vanishing of Z_{rs} at finite N can be understood in this light, for a membrane carrying finite momentum must necessarily be compact and so carry no net central charge.

Ref. [1] discussed a formal method for constructing membrane backgrounds in the $N \to \infty$ limit. The construction uses canonical variables Q, P obeying the commutation relation

$$[Q, P] = \frac{2\pi i}{N} \ . \tag{8}$$

Since this relation cannot be satisfied at finite N, we will instead take the $N \to \infty$ limit in terms of the rescaled variables [6]

$$U_1 = Q\sqrt{Nz_{12}} \ ; \ U_2 = P\sqrt{Nz_{12}} \ . \tag{9}$$

Taking the spectrum of P, Q to go from 0 to 2π, the area of the membrane is $A = (2\pi)^2 N z_{12}$, becoming infinite in the $N \to \infty$ limit. The commutation relation

$$[U_1, U_2] = 2\pi i z_{12} \tag{10}$$

can be represented by differential operators acting on a space of functions. More formally, the N dimensional vector space V_N on which the matrices X act will be replaced by an

[2] with the normalization corresponding to conventions in this paper

infinite dimensional space V_∞ of functions on the membrane worldvolume. The membrane background U_1, U_2 is represented by differential operators acting on V_∞, and fluctuations will be represented by elements of V_∞, *i.e.*, functions on the membrane worldvolume.

A simple representation of $U_{1,2}$ found in the literature consists of taking

$$U_1 = 2\pi i z_{12} \partial_y \quad ; U_2 = y \tag{11}$$

in the $N \to \infty$ limit. As Aharony and Berkooz [6] have pointed out, a cosmetic flaw of this representation is that a membrane carries two spatial coordinates whereas only one is manifest in the above equation.

There exists, however, a more "natural" representation than the one given above. Membranes carry a charge, which in Matrix theory arises from a U(1) subgroup of the U(N) symmetry, or in the continuum limit, the U(1) subgroup of U(∞) which is isomorphic with the infinitesimal area preserving diffeomorphisms of the membrane. Specifically, the membrane charge density $2\pi z_{12}$ is associated with a U(1) magnetic field living on the membrane world volume. In ten dimensions the boosted membrane corresponds to a D2-brane with a large background magnetic field f_{12} on its worldvolume, related to the density σ_0 of D0-branes bound to the D2-brane by [7]:

$$\sigma_0 = \frac{1}{2\pi} f_{12} \ . \tag{12}$$

Since the D0-brane density is

$$\sigma_0 = \frac{N}{A} = \frac{1}{(2\pi)^2 z_{12}} \ , \tag{13}$$

we find that the magnetic field is given by

$$f_{12} = \frac{1}{2\pi z_{12}} \ . \tag{14}$$

Let $\mathbf{a} \equiv (a_1, a_2)$ be the background U(1) vector potential corresponding to the magnetic field: $f_{12} = \nabla \times \mathbf{a}$. We choose

$$a_1 = -\frac{1}{2} f_{12} x_2 \quad , \quad a_2 = \frac{1}{2} f_{12} x_1 \ . \tag{15}$$

Then, a representation of $U_{1,2}$, in which both of the membrane coordinates and the background magnetic field are manifest, is given by

$$\begin{array}{l} U_1 \leftrightarrow 2\pi z_{12}\ (i\partial_{x_1} + a_1(x_1,x_2)) \\ U_2 \leftrightarrow 2\pi z_{12}\ (i\partial_{x_2} + a_2(x_1,x_2)) \end{array} \quad . \tag{16}$$

This representation satisfies the commutation relation (10). To avoid confusion, let us point out that although expressions similar to (16) appear in the context of toroidal compactification [8], here the motivation and the interpretation are quite different. We have not T-dualized anything, the two coordinates x_1, x_2 parametrize the infinite membrane, $(x_1, x_2) \in R^2$.

All other $N \times N$ matrices are represented by functions on the membrane worldvolume:

$$\begin{aligned} \phi_i \quad &\leftrightarrow \quad \phi_i(x_1,x_2) \\ A_0 \quad &\leftrightarrow \quad A_0(x_1,x_2)\ . \end{aligned} \tag{17}$$

(We have suppressed the dependence on the worldvolume time coordinate.) For the components $A_r,\ r = 1,2$ we will choose

$$A_r \leftrightarrow 2\pi z_{12}\ A_r(x_1,x_2)\ . \tag{18}$$

With this normalization, (4) becomes

$$X_r \leftrightarrow 2\pi z_{12}\ [i\partial_{x_r} + a_r(x_1,x_2) + A_r(x_1,x_2)]\ , \tag{19}$$

and we see that $A_\mu(x_1,x_2)$ then has the proper interpretation as the fluctuation around the background (15).

In the $N \to \infty$ limit, the trace operation is replaced by integration over the spatial membrane worldvolume coordinates x_1, x_2:

$$\mathrm{Tr} \leftrightarrow \sigma_0 \int\ dx_1 dx_2 \quad , \tag{20}$$

where the normalization factor σ_0 is included in order to preserve the interpretation of the rank of the matrices as the number of D0-branes $N = Tr\mathbf{1}_N$; we represent the unit matrix $\mathbf{1}_N$ by 1.

Finally, the matrix commutators which involve U_r become commutators of operators and functions, *e.g.*

$$[U_r, A_0] \leftrightarrow (2\pi z_{12})[i\partial_r + a_r(x_1, x_2), A_0(x_1, x_2)] \ , \tag{21}$$

and the matrix commutators which do not involve U_r become commutators of functions, *e.g.*

$$[\phi_i, \phi_j] \leftrightarrow [\phi_i(x_1, x_2), \phi_j(x_1, x_2)] \ . \tag{22}$$

In the case of a single membrane, these will be zero.

Now we can see what happens to L_B. It becomes a 2+1 dimensional Lagrangian density with the terms

$$\begin{array}{lclcr}
F_{0r} & = & \partial_0 A_r + i[U_r, A_0] - i[A_0, A_r] & \leftrightarrow & (2\pi z_{12})\ (\partial_0 A_r(x) - \partial_r A_0(x)) \\
F_{12} & = & -i[U_1, A_2] + i[U_2, A_1] - i[A_1, A_2] & \leftrightarrow & (2\pi z_{12})^2\ (\partial_1 A_2(x) - \partial_2 A_1(x)) \\
D_0\phi_i & = & \partial_0\phi_i - i[A_0, \phi_i] & \leftrightarrow & \partial_0\phi_i(x) \\
D_r\phi_i & = & -i[U_r, \phi_i] - i[A_r, \phi_i] & \leftrightarrow & (2\pi z_{12})\ (\partial_r\phi_i(x))
\end{array}$$

We find

$$\mathcal{L}_B = \mathrm{Tr} L_B \leftrightarrow \sigma_0 \int d^2x\ \frac{T_0}{2}\{-\frac{1}{2}(F_{\mu\nu}F^{\hat{\mu}\hat{\nu}}) + \partial_\mu\phi_i\partial^{\hat{\mu}}\phi_i - 2(2\pi z_{12})^3 F_{12} - (2\pi z_{12})^2\} \ , \tag{23}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the hatted upper indices are raised with the metric

$$\hat{\eta}_{\mu\nu} = \mathrm{diag}\ (1, -\frac{1}{(2\pi z_{12})^2}, -\frac{1}{(2\pi z_{12})^2}) \ . \tag{24}$$

As a check, using the last term we can work out the tension of the membrane to be

$$T_2 = \frac{1}{2\pi} T_0 \tag{25}$$

which is correct. Notice that a key feature of our representation in the $N \to \infty$ limit is the vanishing of commutators such as (22). This is in contrast to the approach in [2], where the authors represented the commutators of matrices by Poisson brackets. Such a representation yields nonvanishing contributions such as

$$\{A_r, A_s\}$$

to the F_{rs} component of the field strength, which is cumbersome from the D2-brane point of view where one expects to obtain a U(1) symmetry. This puzzle seems to be related to the distinction between infinite and finite membranes; for discussion, see [6]. Note also that even if the commutators vanish for a single infinite membrane, we will obtain nonvanishing commutators later when we discuss the case of multiple membrane backgrounds.

It is interesting to examine the relation between the effective action (23) which was derived from Matrix theory, and the analogous Born-Infeld action for a D2-brane in IIA theory. The latter is

$$L_{BI} = -T_2 \int d^2x \sqrt{\det[\eta_{\mu\nu} + F_{\mu\nu} - \partial_\mu \phi_i \partial_\nu \phi_i]} \ , \tag{26}$$

We recall that boosting to the infinite momentum frame in the eleven dimensional description corresponds in ten dimensions to turning on a large magnetic field on the worldvolume of the D2-brane. We therefore make the replacement $F_{12} \to f_{12} + F_{12}$, where $f_{12} = \frac{1}{2\pi z_{12}}$ as in (14), and expand L_{BI} for large f_{12}. To quadratic order in fluctuations we find

$$\begin{aligned} L_{BI} &= -T_2 \int d^2x \left\{ \frac{1}{2\pi z_{12}} + F_{12} \right\} \qquad (27) \\ &- \frac{T_2}{2} \int d^2x \left\{ (2\pi z_{12}) + (2\pi z_{12})^2 F_{12} + \frac{1}{(2\pi z_{12})} \frac{1}{2} F_{\mu\nu} F^{\hat{\mu}\hat{\nu}} - \frac{1}{(2\pi z_{12})} \partial_\mu \phi_i \partial^{\hat{\mu}} \phi_i \right\} \ . \end{aligned}$$

Again, upper indices are raised with the metric (24). To compare with Matrix theory the expression in the first set of braces should be subtracted according to $H_{IMF} = H - p_{11}$. Then, using $T_2 = T_0/2\pi$ and (13), comparison of L_{BI} with (23) shows complete agreement except for a factor of 1/2 multiplying the term linear in F_{12}. This term is a total derivative and does not appear to be significant.

The action (27) represents the starting point for a calculation of Polchinski and Pouliot [9] involving M-momentum transfer between membranes. (Related discussion can be found in [10, 11].) There it was assumed that the action could be obtained directly from Matrix theory; here we have shown explicitly how this can be accomplished.

Consider now a background configuration of k parallel infinite membranes in the transverse directions X_1, X_2. At finite N, this is represented by block diagonal $kN \times kN$ matrices, X_1 has k copies of the "$N \times N$ matrix" Q in the diagonal, and X_2 has k copies of P. Correspondingly, for k infinite membranes we take

$$U_1 \quad \leftrightarrow \quad 2\pi z_{12}(i\partial_1 + a_1) \otimes \mathbf{1}_k \tag{28}$$

$$U_2 \quad \leftrightarrow \quad 2\pi z_{12}(i\partial_2 + a_2) \otimes \mathbf{1}_k \tag{29}$$

where $\mathbf{1}_k$ is the unit $k \times k$ matrix. Thus,

$$[U_1, U_2] = 2\pi i z_{12} \mathbf{1}_k \ . \tag{30}$$

This background breaks the $U(\infty)$ symmetry down to an $U(k)$ symmetry. The fluctuations A_r, ϕ_i are not block diagonal, which would correspond to fluctuations within each brane, but also contain off diagonal components corresponding to strings connecting distinct membranes, thereby accounting for the full $kN \times kN$ matrix structure. The matrices can be represented as a sum of tensor products of $N \times N$ matrices and generators of the $U(k)$ Lie algebra [2, 3]. The $N \times N$ parts, in the $N \to \infty$ limit, are replaced by functions on the membrane - thus (17), (18) are now replaced by

$$\begin{aligned} A_0 \quad &\leftrightarrow \quad A_0^0(x_1, x_2)\ \mathbf{1}_k + A_0^a(x_1, x_2)\ T^a \\ A_r \quad &\leftrightarrow \quad 2\pi z_{12}\ (A_r^0(x_1, x_2)\ \mathbf{1}_k + A_r^a(x_1, x_2)\ T^a) \\ \phi_i \quad &\leftrightarrow \quad \phi_i^0(x_1, x_2)\ \mathbf{1}_k + \phi_i^a(x_1, x_2)\ T^a \ . \end{aligned} \tag{31}$$

where T^a are the traceless generators of the $SU(k)$ Lie algebra. Commutators thus become commutators of fields in the adjoint representation of $U(k)$, *e.g.* (22) becomes

$$[\phi_i(x_1, x_2), \phi_j(x_1, x_2)]_k$$

and the $SU(k)$ part will survive.

The trace over $kN \times kN$ matrices is now represented as follows

$$\mathrm{Tr} \leftrightarrow \sigma_0 \int \ dx_1 dx_2 \mathrm{Tr}_k \ , \tag{32}$$

where Tr_k is the trace over the $k \times k$ structure. Note that the total membrane charge of the multimembrane configuration is

$$
\begin{aligned}
Z_{12} = -\frac{i}{2\pi}\, Tr[X_1, X_2] \quad &\leftrightarrow \quad \sigma_0 \int dx_1 dx_2 z_{12} \mathrm{Tr}_k \mathbf{1}_k \\
&= \quad k \cdot (\text{charge of a single membrane})
\end{aligned}
\tag{33}
$$

as it should be. Plugging everything into the Lagrangian (6), we obtain

$$
\mathcal{L}_B \leftrightarrow \sigma_0 \int d^2x\, \frac{T_0}{2} \mathrm{Tr}_k\, \{-\frac{1}{2}(F_{\mu\nu}F^{\hat{\mu}\hat{\nu}}) + D_\mu\phi_i D^{\hat{\mu}}\phi_i + \frac{1}{2}([\phi_i, \phi_j]_k)^2 - 2(2\pi z_{12})^3 F_{12} - (2\pi z_{12})^2\, \mathbf{1}_k\,\}
\tag{34}
$$

where

$$
\begin{aligned}
F_{0r} &= \partial_0 A_r(x) - \partial_r A_0(x) - i[A_0(x), A_r(x)]_k \\
F_{12} &= \partial_1 A_2(x) - \partial_2 A_1(x) - i[A_1(x), A_2(x)]_k \\
D_0\phi_i &= \partial_0\phi_i(x) - i[A_0(x), \phi_i(x)]_k \\
D_r\phi_i &= \partial_r\phi_i(x) - i[A_r(x), \phi_i(x)]_k
\end{aligned}
$$

We can also check the gauge transformation properties directly. The Matrix Lagrangian (6) is invariant under the $U(kN)$ gauge transformations

$$
\begin{aligned}
\delta A_r &= -i[U_r, \lambda] + i[\lambda, A_r] \\
\delta\phi_i &= i[\lambda, \phi_i] \\
\delta A_0 &= \partial_0\lambda + i[\lambda, A_0]
\end{aligned}
\tag{35}
$$

where the gauge transformation parameter λ (and everything else) is a $kN \times kN$ matrix. In the $N \to \infty$ limit these become

$$
\begin{aligned}
\delta A_r(x) &\leftrightarrow \partial_r\lambda(x) + i[\lambda(x), A_r(x)]_k \\
\delta\phi_i(x) &\leftrightarrow i[\lambda(x), \phi_i(x)]_k \\
\delta A_0(x) &\leftrightarrow \partial_0\lambda(x) + i[\lambda(x), A_0(x)]_k
\end{aligned}
\tag{36}
$$

and give the correct $U(k)$ gauge symmetry of (34).

2.2 Wrapped membrane backgrounds (p=2)

We now compactify the Matrix theory on a two dimensional torus T^2 in the X_1, X_2 directions. This corresponds to compactifying M theory on $T^2 \times S^1$, since one spatial direction was already compactified. We will then consider backgrounds of transverse membranes which wrap around the torus T^2. As a result of the compactification, there will be additional degrees of freedom in the theory, due to strings which wind around the compact directions. As explained by Taylor [8], they can be accomodated in the Matrix theory by replacing each partonic D0-brane by a countably infinite number of copies of it on the noncompact covering space of the torus. The vector space V, on which the matrices act, then has a tensor product structure

$$V = V_N \otimes H^2 \ . \tag{37}$$

The N-dimensional space V_N is multiplied by a countably infinite dimensional space H^2 associated with the countable infinity of copies of D0-branes and strings stretching between them[3].

The description of the Matrix model becomes simpler upon performing T-duality in directions X_1, X_2. The torus T^2 is replaced by its dual torus $\hat{T}^2$, and the 0+1 dimensional quantum mechanics becomes a 2+1 dimensional U(N) super-Yang-Mills theory [8, 1, 13].

Instead of analyzing the fluctuations of a wrapped transverse membrane in the T-dual 2+1 dimensional Matrix theory, we will try to recover a Lagrangian for the massless fluctuations directly from the 0+1 dimensional theory on the initial torus T^2. Our approach will be to construct a membrane on the covering space and then to demand periodicity of the spatial coordinates. We consider a constant density σ_0 of D0-branes on the covering space. The total number of D0-branes is thus infinite, consistent with a nonvanishing value of the membrane charge, $\mathrm{Tr}[X_1, X_2] \neq 0$. The construction then proceeds much as before: we represent the membrane background $U_{1,2}$ as covariant derivatives with a U(1)

[3]Recently, winding supermembranes in eleven dimensions were investigated in [12].

magnetic field, acting on functions of the membrane world volume,

$$\begin{aligned} U_1 &\leftrightarrow 2\pi z_{12}(i\partial_1 + a_1) \\ U_2 &\leftrightarrow 2\pi z_{12}(i\partial_2 + a_2) \end{aligned} \tag{38}$$

where $\mathbf{a} = (a_1, a_2) = -\frac{1}{2} f_{12}(x_2, -x_1)$ as before. However, now the coordinates x_1, x_2 have a finite range:

$$0 \leq x_r \leq L_r \quad , \quad r = 1, 2 \tag{39}$$

where L_1, L_2 are the sizes of the torus T^2, and we demand that all quantities are periodic up to gauge transformations. With this range for the coordinates x_1, x_2, the membrane wraps once around the torus. The trace is represented as in (20), leading to a finite value of total charge

$$Z_{12} = -\frac{i}{2\pi}\,\sigma_0 \int d^2x\ [U^1, U^2] = \sigma_0 z_{12} L_1 L_2 = \frac{L_1 L_2}{(2\pi)^2}\ . \tag{40}$$

for the wrapped membrane.

The fluctuation analysis proceeds as before, and leads to the same effective 2+1 dimensional Lagrangian (23) as before. The tension of the membrane can be checked to be given by the relation (25), as it should.

Consider now 2 parallel membranes wrapped on T^2. In addition to the excitations of open strings within each membrane, there are excitations of strings which interpolate between the two membranes. As discussed in [14], these strings do not have winding modes because they are all homotopic to each other. Effectively, these strings behave as if the spacetime would be non-periodic. However, the homotopy property and the periodicity of the torus is seen in the possibility of introducing Wilson lines which correspond to non-trivial gauge holonomy. We will return to this issue shortly.

For two parallel membranes, we could write

$$\begin{aligned} U_1 &\leftrightarrow 2\pi z_{12}\ (i\partial_1 + a_1) \otimes \mathbf{1}_2 \\ U_2 &\leftrightarrow 2\pi z_{12}\ (i\partial_2 + a_2) \otimes \mathbf{1}_2 \end{aligned} \tag{41}$$

as before, and then proceed exactly as we did in the previous section. This leads to the 2+1 dimensional U(2) Yang-Mills Lagrangian (34). However, it should be possible to make distinction between two singly wound (around T^2) membranes and a *single* membrane winding twice around one of the cycles of T^2. This distinction can be made as follows. We can add constant terms to (41) and define

$$\begin{aligned} U_1 &\leftrightarrow 2\pi z_{12}\left[(i\partial_1 + a_1)\otimes \mathbf{1}_2 + \langle A_1\rangle\right] \\ U_2 &\leftrightarrow 2\pi z_{12}\left[(i\partial_2 + a_2)\otimes \mathbf{1}_2 + \langle A_2\rangle\right] . \end{aligned} \tag{42}$$

The constant terms $\langle A_r\rangle$, $r = 1, 2$ are U(2) Lie algebra valued. This addition does not affect the total 2-brane charge Z_{12}, since the commutator $[\langle A_1\rangle, \langle A_2\rangle]$ is traceless under Tr_2. However, in order to leave the 2-brane charge *density* unaffected, we must require that the $\langle A_1\rangle$ and $\langle A_2\rangle$ commute.

Recall that the fluctuations A_1, A_2 around the background are defined by

$$\begin{aligned} X_1 = U_1 + A_1 &\leftrightarrow 2\pi z_{12}\left[(i\partial_1 + a_1)\otimes \mathbf{1}_2 + \langle A_1\rangle + A_1\right] \\ X_2 = U_2 + A_2 &\leftrightarrow 2\pi z_{12}\left[(i\partial_2 + a_2)\otimes \mathbf{1}_2 + \langle A_2\rangle + A_2\right] . \end{aligned} \tag{43}$$

From these relations it is apparent that the constant terms $\langle A_r\rangle$ can be interpreted as constant backround values of the gauge fields A_r in the U(2) Yang-Mills Lagrangian (34). Thus, they can be used [15] to induce gauge holonomies $\mathcal{U}_r$, through the Wilson lines

$$\mathcal{U}_r = P\exp\{i\oint_r A\}$$

around the cycles of T^2. When the background values are both zero, the holonomies are trivial,

$$\mathcal{U}_1 = \mathcal{U}_2 = \mathbf{1}_2 \ ,$$

around both cycles. From the IIA point of view, we then have a bound state of two D2-branes (and D0-branes), both winding once around the two cycles of T^2. But if we set

$$\begin{aligned} \langle A_1\rangle &= \frac{\pi}{2L_1}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \\ \langle A_2\rangle &= 0 \ , \end{aligned} \tag{44}$$

we get a non-trivial holonomy

$$\mathcal{U}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} .$$

In this case, the result is interpreted as a single D2-brane (with D0-branes) which winds twice around the X_1 cycle of T^2. Both cases yield the same 2-brane charge Z_{12} which is twice the charge (40) of a single membrane. Generalizations to other winding membranes can be obtained in similar manner.

2.3 Longitudinal fivebrane backgrounds (p=4)

The fivebrane of M theory admits a simple description in Matrix theory only if it is wrapped around the longitudinal direction. Since the fivebrane is boost invariant in its world volume directions, going to the infinite momentum frame is more involved than for the transverse membrane. Momentum is instead added by superimposing a gravitational wave solution. However, in the case of a single fivebrane such a configurations necessarily includes nonzero membrane charge as well. To obtain a configuration with vanishing membrane charge one can take two fivebrane solutions with mutually opposite membrane charges and then combine them. In this section we will examine fluctuations around both types of backgrounds. First, let us summarize the backrounds and their properties:

(i) an infinite longitudinal fivebrane with transverse membrane charge density; this reduces in D=10 to a type IIA non-marginal bound state $4+2\perp 2+0$

(ii) infinite longitudinal fivebranes with transverse leftmoving oscillations carrying momentum density in 11th direction; this reduces in D=10 to a marginally bound $4 \parallel 0$ configuration

In case (i), in the $N\to\infty$ limit the background is represented by

$$U_r \leftrightarrow \begin{cases} 2\pi z_{12}\ (i\partial_r + a_r) & r=1,2 \\ 2\pi z_{34}\ (i\partial_r + a_r) & r=3,4 \end{cases}$$

where a_r is a U(1) background field with a field strength $f_{rs} = \partial_r a_s - \partial_s a_r$, with nonzero constant components f_{12}, f_{34} describing magnetic flux densities in 1-2 and 3-4 planes,

$$f_{12} = \frac{1}{2\pi z_{12}} \quad , \quad f_{34} = \frac{1}{2\pi z_{34}} \ .$$

A similar analysis as in Section 2.1 yields an effective Lagrangian for the bosonic fluctuations,

$$\mathcal{L}_B = \sigma_0 \int d^4x \ \frac{T_0}{2}\{-\frac{1}{2}(F_{\mu\nu}F^{\hat{\mu}\hat{\nu}}) + \partial_\mu \phi_i \partial^{\hat{\mu}} \phi_i - f_{rs}F^{\hat{r}\hat{s}} - \frac{1}{2}(f_{rs}f^{\hat{r}\hat{s}})\} \tag{45}$$

where the indices are raised with the metric

$$\hat{\eta}_{\mu\nu} = \mathrm{diag}\ (1, -f_{12}^2, -f_{12}^2, -f_{34}^2, -f_{34}^2) \ , \tag{46}$$

and where σ_0 is the D0-brane density.

In case (ii), the background is represented by covariant derivatives with a selfdual U(2) background[4] field:

$$\begin{array}{ll} a_1 = 0 & a_3 = 0 \\ a_2 = F_0 x_1 \sigma^3 & a_4 = F_0 x_3 \sigma^3 \ . \end{array} \tag{47}$$

The ± 1 diagonal elements of σ^3 represent the superposition of two solutions carrying opposite membrane charge. Note that the background breaks the U(2) symmetry to $U(1)^2$. The field strength $f_{pq} = \partial_p a_q - \partial_q a_p - i[a_p, a_q]$ satisfies the selfduality condition

$$f_{pq} = \tilde{f}_{pq} = \frac{1}{2}\epsilon_{pqrs} \ f_{rs} \ . \tag{48}$$

The transverse volume of the fivebranes becomes infinite in the $N \to \infty$ limit and is given by $A = (2\pi)^4 N z_{1234}$, where z_{1234} is the longitudinal fivebrane charge density. The D0-brane charge density σ_0 is

$$\sigma_0 = \frac{N}{A} = \frac{1}{(2\pi)^4 z_{1234}} \ . \tag{49}$$

It is related to the background field by

$$2\ \sigma_0 = \frac{1}{8\pi^2}\mathrm{Tr}_2\ (f \wedge f) \ , \tag{50}$$

[4]For earlier studies of such background configurations in non-abelian gauge theories, see *e.g.* [16].

which gives

$$F_0 = \frac{1}{2\pi\sqrt{z_{1234}}} \ . \tag{51}$$

Note that σ_0 is the density of D0-branes per fivebrane; the total density of D0-branes is $2\sigma_0$.

We then represent the background (ii) by the covariant derivatives

$$U_r \leftrightarrow 2\pi\sqrt{z_{1234}}\ (i\partial_r \mathbf{1}_2 + a_r) \quad r = 1,2,3,4\ .$$

We can check that the configuration carries a longitudinal 5-brane charge density

$$-\frac{1}{8\pi^2}\epsilon_{pqrs}\mathrm{Tr}[U_p U_q U_r U_s] = 2\cdot z_{1234}\ , \tag{52}$$

where the prefactor 2 represents the presence of two fivebranes. The fluctuations A_0, A_r, ϕ_i about the background are represented by U(2) Lie algebra valued fields in 4+1 dimensions:

$$\begin{aligned} A_0 &\leftrightarrow A_0^0(x)\ \mathbf{1}_2 + A_0^a(x)\ T^a \\ A_r &\leftrightarrow 2\pi\sqrt{z_{1234}}\ (A_r^0(x)\ \mathbf{1}_2 + A_r^a(x)\ T^a) \\ \phi_i &\leftrightarrow \phi_i^0(x)\ \mathbf{1}_2 + \phi_i^a(x)\ T^a\ . \end{aligned} \tag{53}$$

where T^a are SU(2) generators. The effective action for the fluctuations is found to be

$$\mathcal{L}_B = \sigma_0 \int d^4x\, \frac{T_0}{2}\, \mathrm{Tr}_2\, \{-\frac{1}{2}(F_{\mu\nu}F^{\hat\mu\hat\nu}) + D_\mu\phi_i D^{\hat\mu}\phi_i + \frac{1}{2}[\phi_i,\phi_j]^2 - f_{rs}F^{\hat r\hat s} - \frac{1}{2}(f_{rs}f^{\hat r\hat s})\}\ , \tag{54}$$

where the indices are raised with the metric

$$\hat\eta_{\mu\nu} = \mathrm{diag}\ (1, -F_0^2, -F_0^2, -F_0^2, -F_0^2)\ . \tag{55}$$

The energy density of the configuration is the sum of fivebrane plus gravitational wave contributions $H = T + p_{11}$. The infinite momentum frame Hamiltonian is found by subtracting p_{11}: $H_{\mathrm{IMF}} = T$. T is found from the constant term in (54),

$$T = 2T_4 = 2\frac{1}{(2\pi)^2}T_0\ . \tag{56}$$

This is as expected, since two longitudinal fivebranes are interpreted as two D4-branes in the IIA theory.

3 Scattering of branes

Recently, there have been several studies of brane-brane scattering in the context of Matrix theory [6, 17, 18, 19, 20, 21, 22, 23, 24, 9]. These calculations have demonstrated agreement[5] between 11 dimensional supergravity and Matrix theory at one loop and even two loops [22]. For Matrix theory compactified on torii, such calculations amount to loop diagrams in Super-Yang-Mills theory on the dual torus with magnetic backround fields encoding the brane configuration and Higgs field VEVs specifying the kinematics of the scattering problem. From the discussion in Section 2, it should be obvious that a similar correspondence applies also for scattering in uncompactified Matrix theory involving infinite branes.

Note that all the effective actions (23,34,45,54) can be rewritten in a form where the magnetic background fields are combined with the fluctuation fields. Introducing a U(k) gauge field $\mathcal{A}_\mu$,

$$\begin{aligned} \mathcal{A}_0 &= A_0 \\ \mathcal{A}_r &= a_r + A_r \end{aligned} \tag{57}$$

and denoting its field strength by $\mathcal{F}_{\mu\nu}$, the actions (23,34,45,54) can be written in the form

$$\mathcal{L}_B = T_0\sigma_0 \int d^px \ \mathrm{Tr}_k \ \{-\frac{1}{4}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\hat{\mu}\hat{\nu}}) + \frac{1}{2}(\mathcal{D}_\mu\phi_i\mathcal{D}^{\hat{\mu}}\phi_i) + \frac{1}{4}[\phi_i,\phi_j]^2\} \tag{58}$$

where the covariant derivative $\mathcal{D}_\mu$ is taken with respect to the field (57) which includes the background. Now, after including fermions, gauge fixing and ghost terms, we can compute scattering amplitudes from loop diagrams in the appropriate background fields which encode the information about the scattering objects and kinematical setup.

[5]Subtler issues and possible discrepancies have also been discussed, see [25, 26].

3.1 D0 - D6+D0 Scattering

We will perform a scattering calculation involving configurations of D6-branes and D0-branes. Among various other configurations, G. Lifschytz studied the scattering of 6+4+2+0 bound states from D0-branes [18]. (Further discussion can be found in [25].) Recently, W. Taylor [4] showed how to compose configurations of D6 branes and D0-branes which do not carry D4 or D2 brane charges. Such configurations[6] carry a background U(4) gauge field

$$\begin{array}{lll} a_1 = 0 & a_3 = 0 & a_5 = 0 \\ a_2 = F_0 x_1 \mu_1 & a_4 = F_0 x_3 \mu_2 & a_6 = F_0 x_5 \mu_3 \end{array} \tag{59}$$

with traceless U(4) matrices

$$\begin{aligned} \mu_1 &= \mathrm{diag}(1,1,-1,-1) \\ \mu_2 &= \mathrm{diag}(1,-1,-1,1) \\ \mu_3 &= \mathrm{diag}(1,-1,1,-1)\ . \end{aligned} \tag{60}$$

Analogously to (50), the above configuration gives a D0-brane charge density

$$4\ \sigma_0 = \frac{1}{48\pi^3} \mathrm{Tr}_4\ (f \wedge f \wedge f) = 4\ \left(\frac{F_0}{2\pi}\right)^3\ .$$

As before, one can examine the fluctuations about this configuration and obtain an effective U(4) action of the form (58) with $p = 6$ and a metric

$$\hat{\eta}_{\mu\nu} = \mathrm{diag}(1, -F_0^2, -F_0^2, -F_0^2, -F_0^2, -F_0^2, -F_0^2)\ .$$

We have examined the scattering of D0 particles from Taylor's D6+D0 brane configurations and calculated the potential between these objects. The D6+D0 configuration can be understood in the same way as the fivebrane with vanishing membrane charge of Section 2.3. It represents a superposition of four 6+4+2+0 solutions which are chosen to give vanishing D4-brane and D2-brane charges when combined. The scattering calculation that follows reveals this structure, in as much as the potential is found to be four

[6] These configurations are classically stable up to quadratic order. They are thought to be related to the supergravity black hole solutions of [27] carrying 0-brane and 6-brane charges.

times that between a D0 particle and a 6+4+2+0 state. In the Matrix theory 1-loop calculation (which is the same as the 1-loop calculation in the effective theory (58), as we argued above), we encounter the following determinants (the boson, ghost and fermion contributions in the background gauge)

$$\begin{array}{l} \det^{-3}\{(-\partial_\tau^2+H+2c)\ \mathbf{1}_4\}\ \det^{-3}\{(-\partial_\tau^2+H-2c)\ \mathbf{1}_4\} \\ \det^{-2}\{(-\partial_\tau^2+H)\ \mathbf{1}_4\} \\ \det\{(-\partial_\tau^2+H+3c)\ \mathbf{1}_4\}\ \det^{3}\{(-\partial_\tau^2+H+c)\ \mathbf{1}_4\} \\ \det^{3}\{(-\partial_\tau^2+H-c)\ \mathbf{1}_4\}\ \det\{(-\partial_\tau^2+H-3c)\ \mathbf{1}_4\}\ , \end{array} \tag{61}$$

where

$$H = r^2 + c(2(n_1+n_2+n_3)+3)\ .$$

Here r is the distance between the D0-brane and the 6+0 configuration, $c = 1/F_0$, and $n_1, n_2, n_3 = 0, 1, \ldots$. These determinants are the same as those in the calculation by Lifschytz of the D0 - 6+4+2+0 potential, except for additional 4×4 unit matrix factors. The unit matrices signal that the D0-brane sees the four 6+4+2+0 sublayers in the 6+0 configuration, thus the potential between the 6+0 configuration and the D0-brane will be four times that obtained by Lifschytz [18]:

$$V(r) = 4 \cdot \frac{3}{16}\frac{1}{F_0 r}\ . \tag{62}$$

The corresponding supergravity result is easily obtained by considering the 6+0 configuration as a probe moving in the D0-brane background. The calculation is performed in the same manner as by Chepelev and Tseytlin [23], who considered various configurations involving 4-branes and 5-branes. Their paper also collects a number of useful formulae which we will draw from in the following. The effective action for the 6+0 configuration is given by[7]

$$S_6 = \int d^7\xi\ \mathrm{Tr}\{-T_6 e^{-\phi}\sqrt{\det[g_{\mu\nu}\frac{\partial X^\mu}{\partial \xi^i}\frac{\partial X^\nu}{\partial \xi^j} + F_{ij}]} + \mu_6(C_0 F_{12}F_{34}F_{56} + \cdots)\}\ , \tag{63}$$

where $\cdots$ indicates various Chern-Simons terms which are irrelevant for our present purposes. In our units, we have $\mu_6 = T_6$. The field strength in the worldvolume is given

[7]For the effective action we will use Tseytlin's truncated version of the non-abelian Born-Infeld action [28], valid in cases, such as the present one, where the worldvolume field strengths commute.

by $F_{12} = F_0\mu_1, F_{34} = F_0\mu_2, F_{56} = F_0\mu_3$, with $\mu_{1,2,3}$ defined in (60). The spacetime fields to be inserted into S_6 are those induced by a D0-brane source "smeared" in directions $x^1, \ldots, x^6$:

$$\begin{aligned} ds^2 &= H_0^{-1/2}dt^2 - H_0^{1/2}(dx_1^2 + \cdots + dx_9^2) \\ e^{-\phi} &= H_0^{-3/4} \quad ; \quad C_0 \; = \; H_0^{-1} - 1 \\ H_0 &= 1 + \frac{Q_0^{(6)}}{r} \quad ; \quad r^2 \; = \; x_7^2 + x_8^2 + x_9^2 \\ Q_0^{(6)} &= \frac{g}{2}(2\pi)^{5/2} \; . \end{aligned} \tag{64}$$

Substituting into S_6 yields

$$S_6 = -4T_6 \int d^7x \; \{H_0^{-1}(H_0 + F_0^2)^{3/2} - (H_0^{-1} - 1)F_0^3\} \; .$$

To compare with the Matrix theory result we should take $r, F_0 \to \infty$, corresponding to the large distance interaction between objects in the infinite momentum frame of M theory. Then

$$S_6 \approx -4T_6 \int d^7x \; \{\frac{3}{8}\frac{H_0}{F_0} + F_0^3 + \frac{3}{2}F_0\} \; .$$

Now we can read off the potential,

$$V(r) \approx 4 \cdot \frac{3}{8}\frac{T_6 Q^{(6)}}{F_0 r} = 4 \cdot \frac{3}{16}\frac{1}{F_0 r} \; ,$$

in agreement with the Matrix theory result (62).

4 Conclusion

We have seen that by taking the $N \to \infty$ limit of Matrix theory in a particular way, it is possible to recover much of the known physics of D-branes in IIA theory. In particular, by representing the coordinate matrices as covariant derivatives we arrived at the correct Yang-Mills actions governing the low energy dynamics of D-branes. As discussed, these actions are properly interpreted as Born-Infeld actions expanded around large magnetic field backgrounds. This interpretation was verified explicitly, including checking numerical

factors, in the case of a single D2-brane. We also used the effective actions to compute the potential between a D0-brane and a configuration of D6-branes and D0-branes, and saw that the result was in agreement with supergravity.

Finally, although in this paper attention has been restricted to the $N \to \infty$ limit, it would be interesting to investigate D-brane actions at finite N. Despite the absence of infinite BPS branes for such N, it is possible to compare various processes with the predictions of supergravity compactified along a null direction [5, 29]. To understand such processes better, it would be useful see to how much of our techniques may be carried over to finite N.

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